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# Effect of thermophoresis particle deposition on mixed convection from vertical surfaces embedded in saturated porous medium

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# Abstract

**Purpose** – The aim of this paper is to formulate and analyze thermophoresis effects on mixed convection heat and mass transfer from vertical surfaces embedded in a saturated porous media with variable wall temperature and concentration.

**Design/methodology/approach** – The governing partial differential equations (continuity, momentum, energy, and mass transfer) are written for the vertical surface with variable temperature and mass concentration. Then they are transformed using a set of non-similarity parameters into dimensionless form and solved using Keller-box method.

**Findings** – Many results are obtained and a representative set is displaced graphically to illustrate the influence of the various physical parameters. It is found that the increasing of thermophoresis constant or temperature differences enhances heat transfer rates from vertical surfaces and increases wall thermophoresis velocities; this is due to favorable temperature gradients or buoyancy forces. It is also found that the effect of thermophoresis phenomena is more pronounced near pure natural convection heat transfer limit, because this phenomenon is directly temperature gradient- or buoyancy forces-dependent.

**Research limitations/implications** – The predicted results are restricted only to porous media with small pores due to the adoption of Darcy law as a force balance.

**Originality/value** – The paper explains the different effect of thermophoresis on forced, natural and mixed convection heat, and mass transfer problems. It is one of the first works that formulates and describes this phenomenon in a porous media. The results of this research are important for scientific researches and design engineers.

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Keywords Convection, Porous materials, Heat transfer, Fluids, Flow

Paper type Research paper

# Nomenclature

a

b

С

 $C_{\rm f}$ 

 $D^{c_{\mathrm{p}}}$ 

f

- = constant defined in equation (6)
- = constant defined in equation (6)
- = fluid concentration
- = local skin friction factor
- = specific heat capacity
- = Brownian diffusion coefficient
- = dimensionless stream function
- = gravitational acceleration
- h = local heat transfer coefficient
- K = permeability
- k = thermophoresis coefficient
- *Le* = Lewis number,  $\alpha_{\rm m}/D$
- m = constant defined in equation (6)
- $M_{\rm w}(x) = \text{local surface mass flux}$



Ν	= Buoyancy ratio, $[\beta_{\rm C}(C_{\rm w} - C_{\infty})/$	<i>x</i> , <i>y</i>	= axial and normal coordinates	Thermophoresis
	$\beta_{\rm T}(I_{\rm W} - I_{\infty})]$	Cussle	and hala	particle
n	= constant defined in equation (6)	Greek	symbols	1
Nt	= dimensionless temperature ratio, $T_{\infty}/[T_{w}(x) - T_{\infty}]$	$\alpha_{\rm m}$	= effective thermal diffusivity of the porous medium	deposition
$N_{\rm c}$	= dimensionless concentration ratio,	$\beta_{\rm T}$	= coefficient of thermal expansion,	
-	$C_{\infty}/[C_{w}(x) - C_{\infty}]$		$(-1/\rho)(\partial\rho/\partial T)_P$	0.00
Nux	= local Nusselt number. $hx/k$	Bc	= coefficient of concentration	203
P	= pressure	1-0	expansion, $(-1/\rho)(\partial \rho/\partial C)_P$	
$Pe_r$	= local Peclet number, $u_{\infty}x/\alpha$	S	= non-similarity parameter,	
Pr	= Prandtl number, $v/\alpha$		$1/[1 + (Ra_r/Pe_r)^{1/2}]$	
$q_{\rm w}(x)$	= local surface heat flux	η	= pseudo-similarity variable	
Ra	= local Ravleigh number.	$\dot{\theta}$	= dimensionless temperature	
<i>x</i>	$Kg\beta(T_w(x) - T_\infty)x/v\alpha$	$\phi$	= dimensionless concentration	
$Sh_{x}$	= local Sherwood number,	μ	= dynamic viscosity	
50	$M_{\rm w}(x)x/D(C_{\rm w}(x)-C_{\infty})$	υ	= kinematic viscosity	
Т	= temperature	ρ	= fluid density	
и, v	= Darcian velocity components in $x$ -	$ au_{ m w}$	= local wall shear stress	
	and y- directions	ψ	= dimensional stream	
$v_{\rm t}$	= thermophoresis velocity	,	function	
vtw	= thermophoresis velocity at wall			
$V_{t}$	= dimensionless thermophoresis	Subsc	ribts	
c .	velocity. $v_t x / \alpha_m$	W	= surface conditions	
$V_{tw}$	= dimensionless thermophoresis	$\infty$	= free stream condition	
LVV	velocity at wall	t	= thermophoresis effects	
		-		

# 1. Introduction

Thermophoresis is a phenomenon, which causes small particles to be driven away from a hot surface and toward a cold one. Small particles, such as dust, when suspended in a gas temperature gradient, experience a force in the direction opposite to the temperature gradient. This phenomenon has many practical applications in removing small particles from gas streams, in determining exhaust gas particles trajectories from combustion devices, and in studding the particulate material deposition on turbine blades. It has been also shown that thermophoresis is the dominant mass transfer mechanism in the modified chemical vapor deposition process used in the fabrication of optical fiber performance. Also, it is important in view of its relevance to postulated accidents by radioactive particle deposition in nuclear reactors. In many industries, the composition of processing gases may contain any of an unlimited range of particle, liquid, or gaseous contaminants and may be influenced by uncontrolled factors of temperature and humidity. When such an impure gas is bounded by a solid surface, a boundary layer will develop, and energy and momentum transfer gives rise to temperature and velocity gradients. Mass transfer caused by gravitation, molecular diffusion, eddy diffusion, and inertial impact results in deposition of the suspended components onto the surface. In the application of pigments or chemical coating of metals, or removal of particles from a gas stream by filtration, there can be distinct advantages in expositing deposition mechanism to improve efficiency.

Goren (1977) studied the role of thermophoresis of a viscous and incompressible fluid, the classical problem of flow over a flat plate is used to calculate deposition rates and it is found that the substantial changes in surface deposition can be obtained by increasing the difference between the surface and free stream temperatures. Gokoglu and Rosner (1986) and Park and Rosner (1989) obtained a set of similarity solutions for the 2D laminar boundary layers and stagnation point flows, respectively. Chiou (1991) obtained the similarity solutions for the problem of a continuously moving surface in a stationary incompressible fluid, including the combined effects of convection, diffusion, wall velocity and thermophoresis. Garg and Jayaraj (1998) discussed the thermophoresis of small particles in forced convection laminar flow over inclined plates; Epstein et al. (1985) have studied the thermophoresis transport of small particles through a free convection boundary layer adjacent to a cold, vertical deposition surface in a viscous and incompressible fluid. Chiou (1998) has considered the particle deposition from natural convection boundary layer flow onto an isothermal vertical cylinder. Convective flows in porous media have been extensively investigated during the last several decades, due to many practical applications, which can be modeled or approximated as transport phenomena in porous media. Comprehensive literature surveys concerning the subject of porous media can be found in the most recent books by Ingham and Pop (2002), Nield and Bejan (1999), and Pop and Ingham (2001).

Despite the practical importance of thermophoresis there is to our best knowledge, almost one work devoted to the effect of thermophoresis on pure natural convection in a porous medium by Chamkha and Pop (2004), consideration in this work is given to the similarity solutions with approximation of thermophoresis parameter in derivation of governing equations. Therefore, consideration in this work is given to the thermophoresis effects on forced, natural and mixed convection heat, and mass transfer problems from vertical surfaces with variable surfaces temperatures embedded in saturated porous medium (Figure 1). The governing partial differential equations are transformed into non-similar form and then solved using Box method as described by Cebeci and Bradshaw (1984). Numerical results for the velocity, temperature, and



Figure 1. Mixed convection heat and mass transfer over vertical flat plate embedded in fluid saturated porous medium

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concentration profiles as well as the thermophoresis velocity, local coefficient of friction, and local Nusselt number under the effect of different dimensionless groups are presented.

# 2. Mathematical formulation

Consider mixed convection from an impermeable vertical surface embedded in saturated porous medium. The analysis is carried out for the power-law variation of the wall temperature  $T_w(x) = T_\infty + ax^n$  and the power law variation of the wall concentration  $C_w(x) = C_\infty + bx^m$ , where *a* and *b* are constants and *m* and *n* are the exponents. The *x* coordinate is measured from the leading edge of the plate and the *y* coordinate is measured normal to the plate. The gravitational acceleration *g* is acting downward in the direction opposite to the *x* coordinate. The Darcy model which is valid under the conditions of low velocities and small pores of porous matrix is used in the analysis. Also the properties of the fluid are assumed to be constant and the porous medium is treated as isotropic. Allowing for both Brownian motion of particles and thermophoresis transport, the governing equations can be written as (Nield and Bejan, 1999; Chiou, 1998):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u = \frac{Kg}{v} (\beta_{\rm T} (T - T_{\infty}) + \beta_{\rm c} (C - C_{\infty}))$$
<sup>(2)</sup>

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2}$$
(3)

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + \frac{\partial (Cv_t)}{\partial y} = D\frac{\partial^2 C}{\partial y^2}$$
(4)

The first and second terms on the left side of the mass concentration equation is the convective mass flux, while the third term is the thermophoresis mass flux. The *u* and *v* are the Darcian velocity components in *x*- and *y*- directions, respectively, *T* is the fluid temperature, *C* is the fluid concentration, *K* is the permeability of the porous medium, *v* is the kinematic viscosity, *D* is the Brownian diffusion coefficient,  $\alpha_m$  is the effective thermal diffusivity of the porous medium, and  $\beta_T$  and  $\beta_c$  are the thermal expansion coefficient of temperature and concentration, respectively. The effect of thermophoresis is usually prescribed by means of the average velocity, which a particle will acquire when exposed to a temperature gradient. Under boundary layer approximations the temperature gradient in the *y*-direction is very much larger than in the *x*-direction, and therefore only the thermophoresis velocity in the *y*-direction is considered. In consequence the thermophoresis velocity  $v_t$  can be expressed in the form:

$$v_{\rm t} = -k\frac{v}{T} \frac{\partial T}{\partial y} \tag{5}$$

where k is the thermophoresis coefficient. The boundary conditions that describe the governing equations (1)-(5) are:

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$$v = 0, \quad T_{w}(x) = T_{\infty} + ax^{n}, \quad C_{w}(x) = C_{\infty} + bx^{m} \quad \text{at } y = 0$$
  
$$u = u_{\infty}, \quad T = T_{\infty}, \quad C = C_{\infty} \qquad \text{at } y \to \infty.$$
 (6)

Note that n = 0 corresponds to the case of constant wall temperature and m = 0 corresponds to the case of constant wall concentration, while the case of m = n = 1/3 corresponds to the wall with constant heat and mass fluxes.

Equations (1)-(6) can be transformed from the (x, y) coordinates to the dimensionless coordinate  $(s, \eta)$  by introducing the following non-dimensional variables:

$$\eta = \frac{y}{x} P e_x^{1/2} s^{-1}, \quad s = \frac{1}{[1 + (Ra_x/Pe_x)^{1/2}]} \quad \psi = \alpha_m P e_x^{1/2} f(s, \eta) s^{-1},$$
  

$$\theta(s, \eta) = \frac{T - T_{\infty}}{T_{w}(x) - T_{\infty}}, \quad \phi(s, \eta) = \frac{C - C_{\infty}}{C_{w}(x) - C_{\infty}}.$$
(7)

In the equations above, the stream function  $\psi$  satisfied the continuity equation (1) with  $u = \partial \psi / \partial y$  and  $v = -\partial \psi / \partial x$ . Finally, one can obtain the following system of dimensionless equations:

$$f'' = (1 - s)^2 (\theta' + N\phi')$$
(8)

$$\theta'' - nf'\theta + \frac{1}{2}[1 + n(1 - s)]f\theta' = \frac{n}{2}s(1 - s)\left(\theta'\frac{\partial f}{\partial s} - f'\frac{\partial \theta}{\partial s}\right)$$
(9)

$$\frac{1}{Le}\phi'' - mf'\phi + \frac{1}{2}[1 + m(1 - s)]f\phi' + \frac{kPr}{\theta + N_{t}}[\theta'\phi' + (\phi + N_{c})\theta'' - \frac{\phi + N_{c}}{\theta + N_{t}}\theta'^{2}] = \frac{n}{2}s(1 - s)\left(\phi'\frac{\partial f}{\partial s} - f'\frac{\partial \phi}{\partial s}\right)$$
(10)

With the corresponding boundary conditions:

$$f(\boldsymbol{\varsigma}, 0) + 2n\boldsymbol{\varsigma}\frac{\partial f}{\partial \boldsymbol{\varsigma}}(\boldsymbol{\varsigma}, 0) = 0 \text{ or } f(\boldsymbol{\varsigma}, 0) = 0$$
(11)  
$$\theta(\boldsymbol{\varsigma}, 0) = 1, \phi(\boldsymbol{\varsigma}, 0) = 1, f'(\boldsymbol{\varsigma}, \infty) = \boldsymbol{\varsigma}^2, \theta(\boldsymbol{\varsigma}, \infty) = 0, \phi(\boldsymbol{\varsigma}, 0) = 0$$

where,  $Pe_x = u_{\infty}x/\alpha$ ,  $Ra_x = g\beta[T_w(x) - T_{\infty}]Kx/v\alpha$ ,  $Pr = v/\alpha_m$ ,  $Le = \alpha_m/D$ ,  $N = \beta_c(C_w(x) - C_{\infty})/\beta_T(T_w(x) - T_{\infty})$ ,  $N_t = T_{\infty}/[T_w(x) - T_{\infty}]$ ,  $N_c = C_{\infty}/[C_w(x) - C_{\infty}]$  and the primes denotes partial differentiations with respect to  $\eta$ . Note that for the case of s = 0 and  $N_c = 0$ , the governing equations (8)-(10) with the corresponding boundary conditions, equation (11) are reduced to those obtained by Chamkha and Pop (2004) for the case of pure free convection, where a similarity solutions are obtained.

Some of the physical quantities of practical interest include the velocity components u and v in the x- and y- directions, respectively. The wall shear stress  $\tau_w$ , defined as  $\tau_w = \mu (\partial u / \partial y)_{y=0}$ , the local Nusselt number  $Nu_x = hx/k$ , the local Sherwood number  $Sh_x$  and the dimensionless wall thermophoresis deposition velocity  $V_{tw}$ . They are given by:

$$u = u_{\infty} s^2 f'(s, \eta) \tag{12}$$

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$$= -(\alpha_m/x)\left(\frac{1}{2}f - \frac{1}{2}\eta f' + as\frac{\partial f}{\partial s}\right)$$

$$\tau_{\rm w}(x^2/\mu\alpha_m) \left( P e_x^{1/2} + R a_x^{1/2} \right) = f''(\varsigma, 0) \tag{14}$$

$$Nu_{x} \left( Pe_{x}^{1/2} + Ra_{x}^{1/2} \right)^{-1} = -\theta'(\varsigma, 0)$$
(15)

$$Sh_x \left( P e_x^{1/2} + R a_x^{1/2} \right)^{-1} = -\phi'(\varsigma, 0)$$
(16)

and:

$$V_{\rm tw} \left( P e_x^{1/2} + R a_x^{1/2} \right)^{-1} = -\frac{k P r}{1 + N_{\rm t}} \, \theta'(\varsigma, 0) \tag{17}$$

Notice also that for the case of k = 0 (absence of thermophoresis),  $\varsigma = 0$  (pure free convection) the equations are reduced to those of Lai and Kulacki (1991).

#### **3.** Numerical solution

The partial differential equations (8)-(10) under boundary conditions (11) are nonlinear, coupled partial differential equations which posses no closed form solution. Therefore, they must be solved numerically by using an implicit iterative tridiagonal finite-difference method as described by Cebeci and Bradshaw (1984) and Keller (1988). In this method, any quantity g at point  $(\varsigma_n, \eta_i)$  is written as  $g_i^n$  and quantities at the midpoints of grid segments are approximated to second order as:

$$g_j^{n-1/2} = \frac{1}{2} \left( g_j^n + g_j^{n-1} \right), \quad g_{j-1/2}^n = \frac{1}{2} \left( g_j^n + g_{j-1}^n \right)$$
(18)

and the derivatives are approximated to second order as:

v

$$\left(\frac{\partial g}{\partial s}\right)_{j}^{n-1/2} = \Delta s^{-1} \left(g_{j}^{n} - g_{j}^{n-1}\right), \quad \left(g'\right)_{j-1/2}^{n} = \Delta \eta^{-1} \left(g_{j}^{n} - g_{j-1}^{n}\right)$$
(19)

where g is any dependent variable and n and j are the node locations along the s and  $\eta$ directions, respectively. First the second-order partial differential equations are converted into a first order by substitutions f' = s, and  $\theta' = v$ , the difference equations that are to approximate the previous equations are obtained by averaging about the midpoint  $(s_n, \eta_{i-1/2})$ , and those to approximate the resulting equations by averaging about  $(s_{n-1/2}, \eta_{j-1/2})$ . At the first line of s = 0, a system of algebraic equations is obtained and is solved iteratively. The same process is repeated for the next value of  $\varsigma$  and the problem is solved line by line until the desired  $\varsigma = 1$  value is reached. A convergence criterion based on the relative difference between the current and previous iterations is employed. When this difference reaches  $10^{-5}$ , the solution is assumed to have converged and the iterative process is terminated.

The effect of the grid size  $\Delta \eta$  and  $\Delta s$  and the edge of the boundary layer  $\eta_{\infty}$  on the solution had been examined. The results presented here are independent of the grid size and the  $\eta_{\infty}$  at least up to the 5th decimal point. Note that the right-hand terms in

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(13)

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both the momentum, energy and concentration equations, reflect the effect of the nonsimilarity on the mixed convection heat transfer problem under consideration.

The accuracy of the selected method was tested by comparing the results with those of the classical mixed-convection problem over a vertical isothermal impermeable plate (Hsieh *et al.*, 1993). Table I shows a comparison between the Nusselt numbers at different mixed convection parameter,  $\varsigma$  obtained by the presented numerical method and that of the mentioned reference for the case of absence of mass transfer and thermophoresis deposition effects. It is seen that the present results are in a good agreement. This favorable comparison lends confidence in the numerical results to be reported in the next section.

### 4. Results and discussion

The thermophoresis is a phenomenon, which causes small particles to be driven away from a hot surface and to a cold one. The effect of thermophoresis is appeared in the governing equations by the inclusion of N (buoyancy ratio), k (thermophoresis coefficient),  $N_{\rm t}$  (temperature ratio) and  $N_{\rm c}$  (concentration ratio).

In Figures 2-4, the velocity, concentration, and temperature profile are drawn for Pr = 0.72, Le = 10, N = 10,  $N_c = 10$ ,  $N_t = 100$ , k = 0.5, n = m = 0, n = m = 1/3 and different mixed convection parameter s = 0, 0.5, 1. It is obvious that as the mixed convection parameter is increased; the velocity inside boundary layer is increased due to favorable forced convection heat transfer effects and the temperature and concentration gradients are decreased; this leads to higher heat and mass transfer coefficients.

Figure 5 shows wall thermophoresis velocity values for Pr = 0.72, Le = 10, N = 10,  $N_c = 10$ , k = 0.5, n = m = 0, n = m = 1/3 and different values of temperature ratio  $N_t = 10, 20, 50$ . It is clear that as the mixed convection parameter is increased the thermophoresis velocity values are increased due to favorable forced convection heat transfer effects, and it is also clear that the thermophoresis values are decreased when temperature ratios are increased; this is due to small temperature differences between vertical surface and free stream conditions.

Figure 6 shows the local Nusselt number values for Pr = 0.72, Le = 10, N = 10,  $N_c = 10$ , k = 0.5, n = m = 0, n = m = 1/3 and for different values of temperature

		n	= 0.0	n	= 0.5	n = 1.5	
	ζ	Hsieh <i>et al.</i> (1993)	Present study	Hsieh <i>et al.</i> (1993)	Present study	Hsieh <i>et al.</i> (1993)	Present study
	$\begin{array}{c} 0.0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \end{array}$	$\begin{array}{c} 0.4438 \\ 0.4035 \\ 0.3732 \\ 0.3550 \\ 0.3506 \\ 0.3603 \end{array}$	0.4429 0.4062 0.3711 0.3548 0.3510 0.3611	$\begin{array}{c} 0.7704 \\ 0.6991 \\ 0.6419 \\ 0.6026 \\ 0.5844 \\ 0.5890 \end{array}$	0.7710 0.7001 0.6421 0.6032 0.5839 0.5888	1.0000 0.9071 0.8314 0.7783 0.7522 0.7555	0.9980 0.9065 0.8307 0.7779 0.7514 0.7561
t ind to ieh	$0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1.0$	$\begin{array}{c} 0.3832 \\ 0.4174 \\ 0.4603 \\ 0.5098 \\ 0.5642 \end{array}$	$\begin{array}{c} 0.3816 \\ 0.4170 \\ 0.4610 \\ 0.5089 \\ 0.5439 \end{array}$	0.6160 0.6629 0.7259 0.8014 0.8862	$\begin{array}{c} 0.6158 \\ 0.6631 \\ 0.7263 \\ 0.8020 \\ 0.8860 \end{array}$	0.7877 0.8457 0.9250 1.0206 1.1284	$\begin{array}{c} 0.7881 \\ 0.8460 \\ 0.9248 \\ 1.0189 \\ 1.1295 \end{array}$

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# Table I.

Values of  $-\theta'(\zeta, 0)$  at selected values of *n* and for N = 0 compared to those obtained by Hsieh *et al.* (1993)











**Figure 5.** Wall thermophoresis velocity for selected values of temperature ratio parameter  $N_t$  and power exponent *m* and *n* 



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Figure 6. Local Nusselt number values for selected values of temperature ratio parameter  $N_t$  and power exponent m and n

ratio  $N_t = 10, 20, 50$ . It is clear that as mixed convection parameter is increased the Nusselt number values are increased, and the values of variable surface temperature of n = m = 1/3 are higher than those of constant surface temperature of n = m = 0; this is due to excessive heating of the vertical surface, which leads to higher heat transfer rates. The effect of temperature ratio on local Nusselt numbers again to increase them; this is due to favorable temperature gradient between vertical surfaces and the fluid bounded it.

Figure 7 shows the wall thermophoresis velocities for Pr = 0.72, Le = 10,  $N_t = 10$ ,  $N_c = 10$ , n = m = 1/3 and for different values of buoyancy ratio N = 0, 10 and thermophoresis coefficient k = 0.1, 0.2, 0.4, 0.6. The figure shows that as the thermophoresis coefficient is increased the wall thermophoresis velocity is also increased; this is due to favorable temperature gradients. Also the figure shows that as the buoyancy parameter is increased the thermophoresis velocity is increased due to concentration velocities contribution in immigration of fluid particles from the vertical surfaces. It is also clear that as the mixed convection parameter is increased the effect of buoyancy ratio on thermophoresis is decreased due to the absence of natural convection heat transfer effects when  $\varsigma = 1$ . It is also noticed that for the case of N = 0 the thermophoresis coefficient had no effect on thermophoresis velocity.

Figure 8 shows the effect of thermophoresis coefficient on local Nusselt numbers. Again the increasing of k = 0.1, 0.2, 0.4, 0.6 for selected values of Pr = 0.72, Le = 10,  $N_t = 100$ ,  $N_c = 10$ , n = m = 1/3, N = 10 increased local Nusselt numbers because of favorable velocities near vertical surfaces which leads to higher heat transfer rates. Figures 9 and 10 show the effect of buoyancy ratio  $N_c = 10, 20, 50$  on both wall thermophoresis velocity and local Nusselt number values for selected values of Pr = 0.72, Le = 10,  $N_t = 10$ , N = 5.0, k = 0.6, n = m = 0, n = m = 1/3. It is obvious





**Figure 8.** Local Nusselt number values for selected values of thermophoresis parameter *k* and buoyancy parameter *N* 



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Figure 9. Wall thermophoresis velocity for selected values of concentration ratio parameter  $N_c$  and power exponent m and n









that as the buoyancy ratio is increased the thermophoresis wall velocities and local Nusselt numbers are increased; this is due to favorable slip velocities near vertical surfaces. Again as the mixed convection parameter is increased towards the pure forced convection heat transfer limits the effect of buoyancy ratio is decreased; this is due to absence of favorable buoyancy forces beside this limit.

Note that the variation of local Nusselt numbers and dimensionless thermophoresis particle deposition velocity in Figures 6-9 against the mixed convection parameter s reaches a minimum value and then begins to increases toward pure forced convection heat transfer limit; this does not mean that the local values of Nusselt numbers or thermophoresis velocities for mixed convection regime are less than those of pure forced convection or pure natural convection limits. For example, from Table I and for s = 0.8, n = 0, with  $Pe_x = 100$ ,  $Ra_x = 100$ , the values of  $Nu_x/(Pe_x^{1/2} + Ra_x^{1/2})$  is 0.4610 and one finds the values of  $Nu_x = 9.22$  as compared with  $Nu_x = 4.610$  for pure forced convection limit.

Figures 11 and 12 show the effect of buoyancy ratio  $N_c = 10, 20, 50$  on both wall thermophoresis velocity and local Nusselt number values for selected values of Pr = 0.72, Le = 10,  $N_t = 10$ , N = 5.0, for the pure forced and natural convection limits and for different heating and mass exponents n, m. It is obvious that as the buoyancy ratio is increased the thermophoresis wall velocities and local Nusselt numbers are increased; this is due again to favorable slip velocities near vertical surfaces. Again as the power exponents for both heating and mass boundary conditions are increased, both the wall thermophoresis and local Nusselt numbers are enhanced on both limits of pure forced convection and pure natural convection; this is due to excessive heating and excessive concentration difference effects.

# 5. Conclusions

Numerical solutions for heat and mass transfer by steady, laminar boundary layer of a Newtonian fluid over a vertical flat plate embedded in a porous medium in the presence of thermophoresis particle deposition effect were studied. Based on the obtained graphical results, the following conclusions were deduced:

- The thermophoresis and local Nusselt number values are enhanced when temperature difference between surface and free stream conditions are increased; this is due to favorable buoyancy forces.
- When the thermophoresis constant is increased, both thermophoresis velocity and local Nusselt numbers are increased; this is due to favorable velocity mechanism.
- The thermophoresis and local Nusselt number values are decreased when concentration difference between surface and free stream conditions are increased.
- The effect of increasing power heating and mass index *n*, *m* is to enhance thermophoresis wall velocity and local Nusselt numbers; this is due to excessive heating and temperature differences.
- When the buoyancy ratio parameter is decreased towards the zero, the thermophoresis parameter had no effect on both wall thermophoresis and local Nusselt numbers; this is due to small temperature differences between vertical surfaces and free stream condition. Even for large concentration differences the thermophoresis parameter had no effect on both the wall thermophoresis velocity and heat transfer rates for small temperature differences.

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•	The thermophoresis effects on the mixed convection heat transfer problem is
	more pronounced near the pure natural convection heat transfer limit more than
	the forced convection heat transfer limit, this is because the thermophoresis
	mechanism is purely temperature gradient or buoyancy dependent. This leads to
	higher values of Nusselt numbers of pure natural convection heat transfer than
	the corresponding values of pure forced convection heat transfer limit.

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